Lyman-Break Galaxies and the Reionization of the Intergalactic Medium[1]

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ABSTRACT

Near-infrared observations of Lyman-break galaxies at redshifts $z \sim 3$ are beginning to provide constraints on ages, star-formation histories, dust content, metallicities, and stellar masses. At present, uncertainties of more than an order of magnitude are typical for many of these parameters. It is nonetheless interesting to ask what the stellar-population models imply for the existence and luminosities of Lyman-break galaxies at higher redshift. To this end we examine the inferred star-formation rates in two well-studied samples of galaxies as a function of redshift out to $z = 10$ for various best-fit and limiting cases.

Taken at face value, the generally young ages (typically $10^{8\pm0.5}$ yr) of the $z = 3$ Lyman break galaxies imply that their stars were not present much beyond $z = 4$. By $z = 6$ the cosmic star-formation rate $\dot{\rho}_{\rm SFR}$ from the progenitors of these galaxies is less than 10% of $\dot{\rho}_{\rm SFR}$ at $z = 3 \pm 0.5$, even for maximally-old models, provided the derivative of the star-formation rate $SFR(t)$ is monotonic. The escaping Lyman-continuum radiation from such galaxies would be insufficient to reionize the IGM. Thus other sources of ionizing photons (e.g. very massive population III stars) may be needed, and the more normal Lyman-break galaxies

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may be a phenomenon confined to redshifts $z \lesssim 4$. This conclusion changes if $SFR(t)$ was episodic, and we examine the parameters of such bursty star-formation that might be consistent with both the $z = 2 - 4$ luminosity functions and the $z \sim 3$ spectral energy distributions.

Subject headings: galaxies: high-redshift — galaxies: stellar content — galaxies: formation — galaxies: evolution

1. INTRODUCTION

Studies over the past several years have revealed a nascent population of galaxies at redshifts $z = 2 - 4$ with properties that are in many ways similar to those of local starburst galaxies (Giavalisco et al. 1996; Steidel et al. 1996). These galaxies are identified by their strong UV continuum emission and by the presence of strong spectral breaks at Lyman α and the Lyman limit (rest-frame 1216Å and 912Å, respectively). Near-infrared photometry of such Lyman-break galaxies (LBGs) provides access to the rest-frame optical portion of their spectra, and hence greatly improves the constraints on their stellar populations (Sawicki & Yee 1998). Two recent studies (Papovich et al. 2001, hereafter referred to as PDF01; and Shapley et al. 2001) have explored a broad range of stellar population models for LBGs, varying age, star-formation timescales $\tau_{\rm SF}$, metallicities Z, and reddening $E(B-V)$. Our goal in this paper is to examine the implications of these stellar-population models for LBGs at higher redshift. This is a simple thought experiment. In reality we expect star-formation histories to be more complex than these simple models (which, for example, ignore chemical evolution entirely) and we expect galaxy merging to be extremely important at these high redshifts. Nevertheless, the spectral-energy distributions (SEDs) of the individual galaxies at $z \sim 3$ should reflect the products of this evolution, and some of the broad implications for star-formation rates vs. time are relatively insensitive to the details.

One motivation for exploring galaxy evolution at $z > 5$ is to understand the connection between galaxies and the physical conditions in the intergalactic medium (IGM). Observations of QSO absorption lines indicate that the IGM was highly ionized out to redshifts $z \sim 6$, while very recent observations suggest that it was more neutral at higher redshifts (Becker et al. 2001; Djorgovski et al. 2001). It is not known if the sources of ionization were stars or quasars, or whether the stars responsible for the reionization had a mass function at all similar to that observed in the Milky Way. Estimates for the number of ionizing photons needed to reionize the IGM range from 1 to 15 photons per H atom (Madau et al. 1999; Haiman et al. 2001), requiring UV luminosity densities at $z \gtrsim 6$ at least as high as those observed at $z = 3$. If the stellar populations responsible for reionization formed with

an initial mass function (IMF) similar to that observed in present-day stellar populations, then the remnants of these populations must account for a portion of the light emitted by $z = 3$ LBGs. It is thus interesting to explore whether simple models can provide sufficient reionizing photons at $z \sim 6$ without violating the stellar-population constraints for the $z \sim 3$ LBGs.

We review the LBG stellar population constraints in §2. In §3 we turn the clock back on the stellar-population models and compute $\dot\rho_{\rm SFR}(z)$ to higher redshift. In §4 we discuss the implications and in §4.3 we discuss the modifications of $SFR(t)$ or the IMF that might be required to account for both the $z \approx 4$ LBG luminosity function and reionization. Throughout this paper we adopt the cosmological parameters $h, \Omega_{\rm tot}, \Omega_m, \Omega_\Lambda = 0.7, 1.0, 0.3, 0.7$.

2. Lyman-break galaxy stellar populations

Papovich et al. (2001) studied a sample of spectroscopically-confirmed LBGs from the Hubble Deep Field North (HDF) in the redshift range $2.0 \lesssim z \lesssim 3.5$. The data included UV-optical photometry from WFPC2, J and H-band photometry from NICMOS, and $\rm K_s$-band photometry from the KPNO 4m Mayall telescope (Dickinson 1998). Fluxes were determined from profile-weighted photometry, which accounts for the PSF variations and image blending. Stellar-population models from the 2000 version of the Bruzual & Charlot (1993) code were fit to 31 galaxies, varying metallicity, e-folding timescale $\tau_{\rm SF}$, age, IMF (Salpeter, Miller-Scalo, Scalo), extinction, and extinction law (Calzetti et al. 2000, SMC). The geometric mean of the best-fit ages for the sample is 0.12 Gyr for the solar metallicity case. Thus a typical galaxy observed at $z = 3.0$ would have "formed" at $z = 3.15$. Papovich et al. (2001) showed there to be very few galaxies at $z = 3$, even considering those that might have escaped Lyman-break selection, with colors consistent with significantly older ages.

Shapley et al. (2001) analyzed $G, \mathcal{R}, J$, and K_s photometry for a sample of galaxies with spectroscopic redshifts $2.2 < z < 3.4$. Colors were determined from isophotal apertures on PSF-matched images. Solar-metallicity stellar-population models from the 1996 incarnation of the Bruzual & Charlot (1993) code were fit, with various values of $\tau_{\rm SF}$, age, and extinction. The Calzetti (1997) attenuation law was adopted, and the published paper reports results only for the best-fit continuous star-formation models ($\tau_{\rm SF} = \infty$) to the 74 galaxies for which acceptable fits were obtained. The median best-fit age for this sample is 0.32 Gyr, implying a formation redshift $z = 3.4$ for a typical galaxy observed at $z = 3$.

Clearly, the inferred ages for *monotonic* star-formation histories in these two studies are very young. Papovich et al. (2001) also found that a substantial fraction of the stellar mass

could be hidden in a "maximally old" passively evolving population that formed instantaneously at $z = \infty$. Inferred LBG masses typically increase by a factor of 3 in such models. For our purposes, the interesting point is that the star-formation rate appears unlikely to have been constant over a Hubble time — i.e. M_s/t_H, the stellar mass divided by the Hubble time at the LBG redshift, is typically much less than the measured SFR at $z \sim 3$. This generic conclusion is unlikely to be very sensitive to the details of the stellar-population models.

3. Turning back the clock

In exploring the implications of these models, we shall consider three limiting cases: (*i*) a single burst of star formation, (*ii*) continuous star formation starting at some time t, and (*iii*) a two-burst model. Multiple burst models would be intermediate between these cases. Figure 1 shows the star-formation rate as a function of redshift inferred for each galaxy in the PDF01 sample for solar-metallicity models. Models with $0.2Z_\odot$ give younger ages and higher star-formation rates. The top two panels of Figure 1 show single-burst models with $SFR \propto e^{-t/\tau_{\rm SF}}$ and continuous star-formation models. In the burst models only one out of the 31 galaxies would have been present at $z = 6$. In the oldest continuous-star-formation models, six out of 31 or 19% would have been present at $z = 6$. Figure 1c shows the results for the best-fit solar-metallicity continuous-star-formation models of Shapley et al. (2001). The models imply that only 17% of the galaxies were present at $z = 6$.

Models of type (*iii*) with two distinct episodes of star formation allow more star formation at higher redshift. Papovich et al. (2001) fit maximally-old models to their LBG sample, deriving constraints on the mass of an old population that formed with a Salpeter IMF in an instantaneous burst at $z = \infty$. This model quantifies how much stellar mass can be hidden "underneath the glare" of the younger population. The star-formation rate predicted at $z = 6$ from such maximally old components is zero, because all star-formation happened at higher redshift. Starbursts induced by mergers are likely to be spread out over a range of redshift. If the older burst in the LBGs is put at redshift lower than $z = \infty$, the mass in the burst must be lower. Rather than fit a whole suite of models of different burst redshifts, we can, to a good approximation, scale the allowable mass in the old component by a power-law fading model. By fitting the B-band luminosity vs. time for $10^7 < t < 2 \times 10^9$ yr, we find $L_{\rm B} \propto t^{-0.8}$ for a Salpeter IMF for an instantaneous burst in the Bruzual & Charlot solar-metallicity models. The B band is chosen because the older burst population contributes mostly longward of $\lambda_{\rm rest} = 3000$ Å (See PDF01, Fig. 19). This fading exponent is slightly shallower than that adopted by Hogg & Phinney (1997), because

of the narrower age range used for our estimate. A Scalo IMF would fade more gradually, as would a lower-metallicity model. The allowed mass in a burst as a function of age is $M(z) = M_{\rm max}({\rm age}/t_H)^{0.8}$, where $M_{\rm max}$ is the maximum mass allowed in an instantaneous burst formed at $z = \infty$. If each galaxy had an instantaneous probability $P(z)$ of forming stars in a burst of typical duration δt, then the average SFR per galaxy from an ensemble of such galaxies would be $M(z)P(z)/\delta t$. For simplicity we adopt a constant $P(z)$ from $z = 10$ to the observed LBG redshift $z_{\rm obs}$. (We consider varying $P(z)$ in §4.3.) The ensemble-average star-formation rate is thus $\xi(z) = M(z)/(t_{\rm obs} - t_{10})$, where t_{10} is the age of the universe at $z = 10$, and $t_{\rm obs}$ is the age of the universe at the redshift of the LBG. In the current generation of semi-analytic hierarchical models, the rate of star-formation due to mergers decreases at $z > 3$ (Cole et al. 2000; Somerville et al. 2001). Therefore our assumption of constant $P(z)$ puts a higher proportion of star-formation at high redshift.

Figure 1d shows the SFR vs. redshift implied by such a stochastic model for two individual galaxies in the PDF01 sample. The low-redshift spikes in the star-formation rate correspond to the young component that dominates the light at the observed redshift; the star-formation progressing to higher redshift represents the mean for an ensemble of stochastic bursts. Obviously any single galaxy would simply show two spikes of star formation for this kind of model, but if we consider such a galaxy as a proxy for millions of others, the star-formation history shown in the figure represents the maximal rate of star-formation due to stochastic bursts as a function of redshift.

The results become clearer if we consider the entire sample of galaxies. Figure 2 shows the evolution of $\dot\rho_{\rm SFR}(z)$ with time relative to that $z = 3$ computed by summing up the models shown in the previous figures. The top panel shows the monotonic star-formation histories. For these cases the inferred co-moving density of star formation declines dramatically from $z = 3$ to higher redshift. The stochastic burst model is shown by the solid curve in Fig. 2b. Even if we put the maximum mass allowed in stochastic-starbursts at redshifts $z > z_{\rm observed}$, the star-formation rate at $z = 6$ is still a factor of 3 below that at $z = 3$.

4. Implications

4.1. The Luminosity Function at $z \sim 4$

All of the star-formation histories considered so far imply a dramatic decline in star formation rate by $z = 4$. However the observed LBG rest-frame UV luminosity functions are very similar at $z = 3$ and $z = 4$, and the integrated star-formation rates derived therefrom differ only by a factor of 1.1 ± 0.4 (Steidel et al. 1999). Thus the star-formation *histories*

derived from the $z = 3$ LBGs are in direct conflict with the star-formation *rates* derived for the $z = 4$ LBGs.

4.2. Reionization

If all of the ionizing photons come from star formation, Madau et al. (1999) estimate that the amount of star-formation needed is

$$\dot{\rho}_{\rm SFR} \approx 0.013 {f_{\rm esc}}^{-1} (\frac{1+z}{6})^3 (\frac{\Omega_b h_{50}^2}{0.08})^2 C_{30}\, M_\odot\, {\rm yr}^{-1} {\rm Mpc}^{-3}, \quad (1)$$

where $f_{\rm esc}$ is the mean fraction of Lyman-continuum radiation that escapes from galaxies, Ω_b is the baryon density, h_{50} is the Hubble constant in units of $50\,{\rm km\,s^{-1}Mpc^{-1}}$, and $C_{30} = 30\langle n_{\rm HII}^2\rangle/\bar{n}_{\rm HII}^2$ is the ionized hydrogen clumping factor. Adopting $f_{\rm esc} = 0.1$, the required density of star-formation for reionization in this model is a factor of 1.3 times higher than the dust-corrected $\dot{\rho}_{\rm SFR}$ at $z \sim 3$ measured by Steidel et al. (1999). [3] In contrast, the star-formation rates inferred from the SED fits imply a sharp decrease in $\dot{\rho}_{\rm SFR}$ between $z = 3$ and $z = 6$. For the monotonic star-formation histories, this decrease is at least one order of magnitude. Even for the case of stochastic bursts the star-formation rate is still well below that needed for reionization. The problem becomes even more severe if a significant fraction of the baryons are already collapsed into minihalos at the time of reionization. In this case the required number of ionizing photons increases by a factor of 10-20 (Haiman et al. 2001), and all models fall short even if $f_{\rm esc} = 1$.

4.3. What Kind of Starbursts are Needed?

In the discussion above, we adopted a uniform starburst probability $P(z)$ and found that such a model was unable to produce enough photons at $z \gtrsim 6$ to account for reionization (while at the same time fitting $z = 3$ LBG SEDs). One simple modification would be to increase the burst probability at high redshift. Keeping $P(z)$ uniform, we require that bursts occur with uniform probability over the redshift range $z_{\rm min} < z < z_{\rm max}$ and vary $z_{\rm min}$ and $z_{\rm max}$ until the SFR at $z = 6$ equals that at $z = 3$. Independent of $z_{\rm max}$ we find that values

[3] The value of $f_{\rm esc}$ is highly uncertain. Measurements by Steidel et al. (2001) give a flux ratio $F(900\mathring{A}/F(1500\mathring{A}) \approx 0.2$ for a sample of galaxies $z \approx 3.4$ (but see Giallongo et al. (2002)). This is higher than most stellar population models predict even if $f_{\rm esc} = 1$. On the other hand, the estimated mean dust attenuation factor for Lyman break galaxies is 4.4 at $\lambda = 1500\,\mathring{\rm A}$ (Steidel et al. 1999), implying $f_{\rm esc} \lesssim 0.2$ even ignoring neutral hydrogen opacity.

of $z_{\rm min} > 4.4$ are required to achieve this. Thus, LBG evolution would be characterized by an early epoch of star-formation responsible for reionization, followed by a lull, followed by increased star formation at $z \sim 3$. This kind of behavior might be caused by reheating of the IGM during reionization (Cen & McDonald 2001). However, such a scenario would increase the discrepancy at $z = 4$.

More star formation can be hidden in bursts if the bursts fade faster. For a first-order estimate, we adopt a power-law fading model $L(t) \propto t^{-\zeta}$. For a Salpeter IMF in the B band $\zeta = 0.8$. We vary ζ until the UV luminosity density at $z = 6$ equals that at $z = 3$. We find that a fading exponent $\zeta = -1.1$ is required. As shown by the dashed curve in Fig. 2b, such a model still falls short of the observed luminosity-density at $z = 4$, but is within the uncertainties. If the IMF is a powerlaw $\phi(M)dM \propto M^{-(1+x)}$, a fading exponent $\zeta = -1.1$ requires an IMF slope $x = 0.5$ compared to the Salpeter value $x = 1.35$ (for an instantaneous-burst solar-metallicity stellar population). A steeper fading slope $\zeta = -1.2$ (corresponding to an IMF slope $x = 0.3$) is needed to bring $\dot{\rho}_{\rm SFR}$ at $z = 4$ to within a factor of 1.3 of that at $z = 3$. Lower metallicities require even more top-heavy IMFs. Options other than varying the IMF are of course possible (e.g. evolved stellar populations could be hidden by dust that builds up over timescales of 10^8 to 10^9 yrs). However, the requirement for faster-than-Salpeter fading is robust. Furthermore, the fading must be even faster if galaxies on average have more than two burst episodes.

5. Conclusion

In summary, we find that the monotonic star-formation histories that best match $z = 3$ LBG spectra fail (by a large factor) to provide enough photons to reproduce the luminosity density at $z = 4$ or to reionize the IGM at $z \gtrsim 6$. Even stochastic-burst models, which permit factors of $3 - 10$ more mass to be formed at higher redshift, fail to resolve the shortfall. We are left with a variety of more complex alternatives.

(1) If we require that the stellar populations responsible for reionization formed with typical Galactic IMF ($x \sim 1.35$), and that such star formation did not show a pronounced gap between $z = 6$ and $z = 3$, then we must conclude that the remnants of the stellar populations responsible for reionization *do not reside in* $z = 3$ *Lyman-break galaxies*. This is possible, for example, if undetected dwarf galaxies with number-densities higher than the extrapolation of the LBG luminosity function dominate the ionizing background.

(2) The spectral energy distributions of $z = 3$ LBGs allow for a separate epoch of normal-IMF star formation at very high redshift provided that such star formation ceased

by $z \approx 6$, leaving a gap in star-formation until $z \lesssim 4$. This solution to the reionization problem glosses over the need to explain the $z = 4$ LBG luminosity function.

(3) Reionization could have been caused by stellar populations heavily weighted toward massive stars (Larson 1998; Abel et al. 2000; Oh et al. 2001). If this phenomenon was confined to high redshift (e.g. high-mass, zero-metallicity Population III stars), then the remnants could reside in lower redshift LBGs as black holes or neutron stars. This solution to the reionization problem also fails to solve the $z = 4$ LBG problem.

(4) Both problems can be resolved if the star formation in LBGs was episodic and the stars formed with a top-heavy IMF. Bursts of star formation associated with mergers are a natural consequence of hierarchical models of galaxy formation, and are incorporated to varying degrees into many of the current semi-analytical models (Kauffmann & Haehnelt 2000; Cole et al. 2000; Somerville et al. 2001). With the assumption that the starburst probability $P(z)$ is constant over $3 \lesssim z < 10$, we find that an IMF slope $x \sim 0.3 - 0.5$ would be required to explain both the relative constancy of the LBG luminosity function over the range $2 < z < 4.5$ and plausibly provide enough star formation at $z \gtrsim 6$ to reionize the IGM. Top-heavy IMFs could in principle result from higher ISM pressure during mergers (Padoan et al. 1997; Chiosi et al. 1998; but see Scalo et al. 1998). Local tests are difficult to carry out because the remnants of the massive stars responsible for producing the UV photons at high redshift are neutron stars or black holes today. In the Galactic bulge, the best fit slope for the mass-function for $M < 1 M_\odot$ is $x = 0.33$ (Zoccali et al. 2000). Micro-lensing experiments (Udalski et al. 1994; Alcock et al. 1997) do not yet rule out the possibility that this slope could have continued up to $100 M_\odot$. More direct constraints on the star-formation histories of LBGs will improve greatly over the next few years with the advent of the Space Infrared Telescope Facility and the Advanced Camera for Surveys and ultimately the Next Generation Space Telescope.

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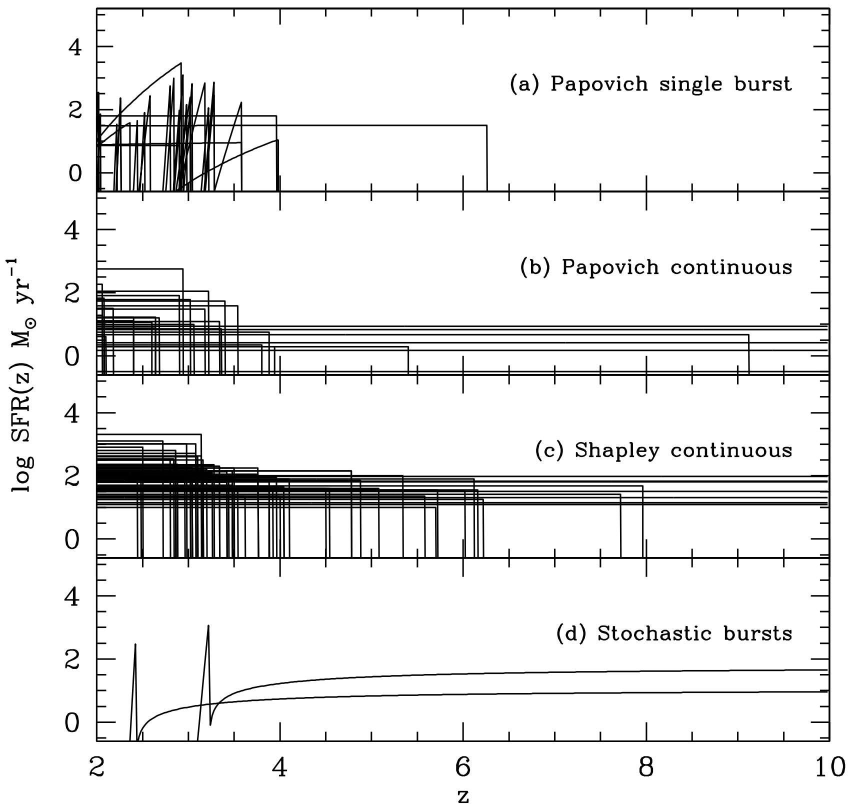

Fig. 1.— Star-formation rate vs. time for individual galaxies, as inferred from the SED models. The top panel shows the best-fit models of type (*i*) described in the text from Papovich et al. (2001). Panel (b) shows the star-formation histories from models of type (*ii*) characterized by a stellar mass M and an age, with a constant star-formation rate once the galaxy has formed. The models shown here are the oldest ones consistent with the SEDs within the 95% confidence interval. Panel (c) shows models with continuous star-formation using the best-fit parameters for the Shapley et al. (2001) sample. Panel (d) shows two examples of the stochastic burst model described in the text applied to galaxies 97 and 1115 in the PDF01 sample.

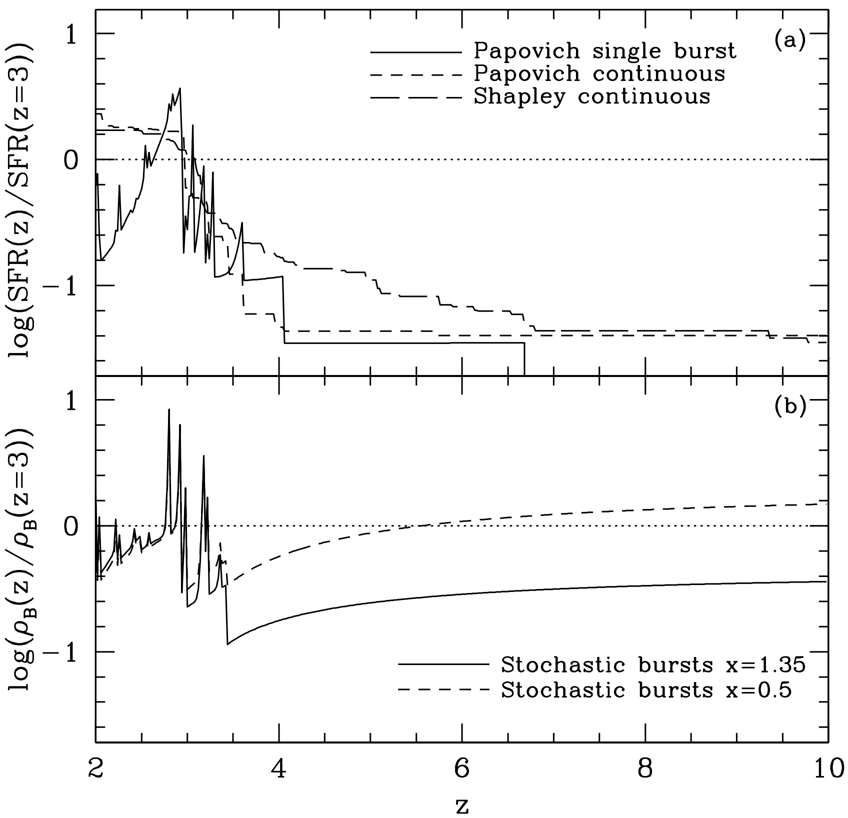

Fig. 2.— Top panel — star-formation density vs. time for the monotonic models, normalized to the mean in the range at $2.5 < z < 3.5$. The solid curve is for the PDF01 single-burst models. The short-dashed curve is for their continous star-formation models. The long-dashed curve is for the Shapley et al. (2001) continuous star-formation models. Bottom panel — rest-frame B-band luminosity density vs. time for the stochastic burst models with a Salpeter IMF (solid) and a top-heavy IMF with $x = 0.5$ (dashed). For fixed IMF in the stochastic-burst model the B-band luminosity density roughly scales with the star-formation rate. As the IMF is varied the zeropoint of this scaling changes, so it is more relevant to consider luminosity densities. The B-band is shown because that is what we calculate from the power-law fading model. The UV luminosity density is more relevant for the discussion of reionization and the $z = 4$ luminosity function. Even for an extreme IMF slope of $x = 0.3$, the $m(1500\text{Å}) - B$ colors and $m(860\text{Å}) - B$ colors are within 0.2 and 0.3 mag, respectively, of the colors for the Salpeter IMF.